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CERN-PH-TH/2008-192

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Abstract

We provide a study of the supersymmetric Adler–Bardeen anomaly in the $\mathcal{N}=1,\ d=4,6,10$ super-Yang–Mills theories. We work in the component formalism that includes shadow fields, for which Slavnov–Taylor identities can be independently set for both gauge invariance and supersymmetry. We find a method with improved descent equations for getting the solutions of the consistency conditions of both Slavnov–Taylor identities and finding the local field polynomials for the standard Adler–Bardeen anomaly and its supersymmetric counterpart. We give the explicit solution for the ten-dimensional case.

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1 Introduction

The gaugino of even-dimensional $\mathcal{N} = 1$ supersymmetric Yang–Mills theories is a chiral spinor. This implies the existence of an Adler–Bardeen one-loop anomaly. Its effect is made manifest by the non-vanishing of a relevant form factor of well defined one-loop amplitudes, as predicted by the consistency equations [1] and by their solution given by the Chern–Simons formula [2, 3, 4]. It is of course well-known that the existence of anomalies in ten-dimensional supersymmetric Yang–Mills theory has triggered fundamental progress in string theories [5, 6]. The consistency of these anomalies with $\mathcal{N} = 1$ supersymmetry eluded however a complete analysis. In this paper, we address this problem using recent progress in the formulation of supersymmetric theories in component formalism and in various dimensions.

As a matter of fact, in a supersymmetric gauge theory, the Wess and Zumino consistency conditions must be generalized in order to be compatible with supersymmetry. The standard Adler–Bardeen anomaly must come with a supersymmetric counterpart. The method of [2, 3, 4] was however generalized to higher dimensions both in component formalism [7] and in superspace [8] to determine this supersymmetric counterpart, but no explicit expression was given for the ten-dimensional case. Such an expression was derived later in [9] for the coupled $\mathcal{N} = 1$ supergravity and super-Yang–Mills theory for the supersymmetrization of the Green–Schwartz mechanism [6].

On the other hand, recent results in component formalism based on the introduction of shadow fields [10] have allowed for the definition of two independent Slavnov–Taylor identities. This has permitted the disentangling of gauge invariance and supersymmetry. In this way, one gets a consistent analysis of the compatibility of the Adler–Bardeen anomaly with supersymmetry, and an algebraic proof was given for the absence of anomalies in $\mathcal{N} = 2, 4,\ d = 4$ super-Yang–Mills theories and for the fact that in the case of $\mathcal{N} = 1$ the only possible anomaly is of the Adler–Bardeen type.

The purpose of this paper is to give a systematic way for solving the supersymmetric consistency equations for the supersymmetric Adler–Bardeen anomaly in the cases of $\mathcal{N} = 1,\ d = 4, 6, 10$ super-Yang–Mills theories, following the same logic as that of [10] and completing it by determining the explicit expression for the ten-dimensional case.

2 Adler–Bardeen anomaly in super-Yang–Mills theories

We first introduce some definitions that apply to $\mathcal{N} = 1$ supersymmetric Yang-Mills theories in general and focus on the ten-dimensional case afterwards. Let s be the BRST operator associated to ordinary gauge symmetry and Q the differential operator that acts on the physical fields as an ordinary supersymmetry transformation minus a gauge transformation of parameter a scalar field c, that is $Q \equiv \delta^{Susy} - \delta^{gauge}(c)$. The shadow field c allows for the elimination of the field dependent gauge transformations in the commutators of the supersymmetry algebra [10]. It completes the usual Faddeev–Popov ghost Ω associated to BRST symmetry. The s and Q operators verify

$$s^2 = 0\,, \quad \{s\,, Q\} = 0\,, \quad Q^2 \approx \mathcal{L}_\kappa \tag{1}$$

where $\approx$ means that this relation can hold modulo the equations of motion and κ is the bilinear function of the supersymmetry parameter, $\kappa^\mu = -i(\epsilon\gamma^\mu\epsilon)$. In addition to the ghost number, we assign a shadow number, equal to one for the supersymmetry parameter and for the shadow field c, and zero for the other fields. The Q operator increases the shadow number by one unit. Each field and operator has a grading determined by the sum of the ghost number, shadow number and form degree. Transformation laws for the various fields can be deduced from the definition of an extended curvature $\tilde{F}$, by decomposition over terms of all possible gradings of the following horizontality condition

$$\tilde{F} \equiv (d + s + Q - i_\kappa)\big(A + \Omega + c\big) + \big(A + \Omega + c\big)^2 = F + \delta^{Susy} A \tag{2}$$

where A is the gauge connection and $F = dA + AA$. At the quantum level, one introduces sources for the non-linear s, Q and $s\,Q$ transformations of all fields. The BRST invariant gauge-fixed local action with all needed external sources is then given by

$$\Sigma = S[\varphi] + s\,\mathbf{\Psi} + S_{\text{ext}} \tag{3}$$

The BRST and supersymmetry invariances of Σ imply both Slavnov–Taylor identities

$$\mathcal{S}_{(s)}(\Sigma) = 0\,, \quad \mathcal{S}_{(Q)}(\Sigma) = 0 \tag{4}$$

where $\mathcal{S}_{(s)}$ and $\mathcal{S}_{(Q)}$ are the Slavnov–Taylor operators associated to the s and Q operators, respectively [1]. These identities imply the following anticommutation relations between

[1] We refer to [10] for more explicit definitions.

the associated linearized Slavnov–Taylor operators $\mathcal{S}_{(s)|\Sigma}$ and $\mathcal{S}_{(Q)|\Sigma}$

$$\mathcal{S}_{(s)|\Sigma}^{\ 2} = 0\,, \quad \{\mathcal{S}_{(s)|\Sigma}\,, \mathcal{S}_{(Q)|\Sigma}\} = 0\,, \quad \mathcal{S}_{(Q)|\Sigma}^{\ 2} = \mathcal{P}_\kappa \tag{5}$$

where $\mathcal{P}_\kappa$ is the differential operator that acts as the Lie derivative along κ on the fields and external sources [2]. An anomaly is defined as an obstruction – at a certain order n of perturbation – to the implementation of the Slavnov–Taylor identities on the vertex functional $\Gamma = \Sigma + O(\hbar)$, that is

$$\mathcal{S}_{(s)}(\Gamma) = \hbar^n \mathscr{A}\,, \quad \mathcal{S}_{(Q)}(\Gamma) = \hbar^n \mathscr{B} \tag{6}$$

where $\mathscr{A}$, and $\mathscr{B}$ are respectively integrated local functionals of ghost number one and shadow number one, defined modulo $\mathcal{S}_{(s)}$- and $\mathcal{S}_{(Q)}$-exact terms. The introduction of the linearized Slavnov–Taylor operators permits one to write the consistency conditions

$$\mathcal{S}_{(s)|\Sigma}\mathscr{A} = 0^{(2,0)}\,, \quad \mathcal{S}_{(Q)|\Sigma}\mathscr{A} + \mathcal{S}_{(s)|\Sigma}\mathscr{B} = 0^{(1,1)}\,, \quad \mathcal{S}_{(Q)|\Sigma}\mathscr{B} = 0^{(0,2)} \tag{7}$$

where the superscripts (g, s) denote the ghost and the shadow number. Due to (5), the problem of the determination of the solutions to these conditions is a cohomological problem. The consistent Adler–Bardeen anomaly is thus defined as the pair $\mathscr{A}$ and $\mathscr{B}$, identified as the elements $(1,0)$ and $(0,1)$ of the cohomology of the operators $\mathcal{S}_{(s)|\Sigma}, \mathcal{S}_{(Q)|\Sigma}$, in the set of integrated local functionals depending on the fields and sources. It can be shown that the cohomology of the linearized Slavnov–Taylor operators in the set of local functionals depending on the fields and sources is completely determined by that of the classical operators in the set of local functionals depending only on the fields, provided such functionals are identified on the stationary surface, i.e., modulo equations of motion [11, 10]. We will thus consider the consistency conditions

$$s\,\mathscr{A} = 0^{(2,0)}, \quad Q\mathscr{A} + s\,\mathscr{B} = 0^{(1,1)}, \quad Q\mathscr{B} = 0^{(0,2)} \tag{8}$$

To determine the solutions of these equations in $d = 2n - 2$ dimensional space-time, we formally define the Chern character $2n$-form $\mathrm{Ch}_n \equiv \mathrm{Tr}\ \tilde{F}^n$, where $\tilde{F}$ has been introduced in Eq. (2) [3]. From a generalization of the algebraic Poincaré lemma and the Chern–Simons identity, Ch_n can locally be written as a $(\tilde{d} \equiv d + s + Q - i_\kappa)$-exact term

$$\mathrm{Tr}\ \tilde{F}^n = \tilde{d}\,\mathrm{Tr}\ W_{2n-1}(\tilde{A}, \tilde{F}) \tag{9}$$

[2]The fact that Q^2 is a pure derivative only modulo the equations of motion on the gaugino of the ten-dimensional case is solved for the linearized Slavnov–Taylor operator $\mathcal{S}_{(Q)|\Sigma}$ by introducing suitable source terms in (3).

[3]The following procedure is actually valid for any invariant symmetric polynomial, which covers the case of so-called factorized anomalies.

W_{2n-1} is the Chern–Simons form, which can be calculated form the formula

$$W_{2n-1}(\tilde{A}, \tilde{F}) = n \int_0^1 \mathrm{d}t \, \mathrm{Tr} \, (\tilde{A} \, \tilde{F}_t^{n-1}) \tag{10}$$

where $F_t = tdA + t^2 A^2$. The term with grading $(2,0)$ in (9) gives the standard Adler–Bardeen anomaly [4]

$$\mathscr{A} \equiv \int W^{(1,0)}_{2n-2} \, , \qquad s \, \mathscr{A} = 0 \tag{11}$$

The term with grading $(1,1)$ gives a solution for the consistency condition

$$Q\mathscr{A} + s \, \mathscr{B}^{\mathrm{c}} = 0 \tag{12}$$

which is given by

$$\mathscr{B}^{\mathrm{c}} \equiv \int W^{(0,1)}_{2n-2} \tag{13}$$

However, we have not yet a solution to the consistency equations, since the term Tr $\tilde{F}^n$ with grading $(0,2)$ gives a breaking of the consistency condition $Q\mathscr{B}^{\mathrm{c}} = 0$, according to

$$Q\mathscr{B}^{\mathrm{c}} = \tbinom{n}{2} \int \mathrm{Tr} \; \delta^{Susy} A \; \delta^{Susy} A \; F^{n-2} \tag{14}$$

where $\tbinom{n}{2}$ stands for the binomial coefficient.

The solution of this problem can be solved as follows. One observes that $\mathscr{B}^{\mathrm{c}}$ in Eq. (14) is a particular solution of Eq. (12), so that one can add to it a local functional of the fields $\mathscr{B}^{\mathrm{inv}}$, provided their sum is Q-invariant. To preserve the condition (12), $\mathscr{B}^{\mathrm{inv}}$ must be s-closed. But since $Q\mathscr{B}^{\mathrm{c}}$ is not s-exact and since $\{ s \, , Q\} = 0$, no s-exact element of $\mathscr{B}^{\mathrm{inv}}$ can contribute and $\mathscr{B}^{\mathrm{inv}}$ must be in the cohomology of s. Therefore, the consistency conditions (8) are fulfilled provided there exists a gauge-invariant local functional of the physical fields satisfying

$$\delta^{Susy} \mathscr{B}^{\mathrm{inv}} = -\tbinom{n}{2} \int \mathrm{Tr} \; \delta^{Susy} A \; \delta^{Susy} A \; F^{n-2} \tag{15}$$

so that

$$\mathscr{B} = \mathscr{B}^{\mathrm{c}} + \mathscr{B}^{\mathrm{inv}} \, , \quad Q\mathscr{B} = 0 \tag{16}$$

We now address the problem of determining $\mathscr{B}^{\mathrm{inv}}$. We keep general d-dimensional notations, since we have in mind the cases of $\mathcal{N} = 1$ super-Yang–Mills theories in $d = 4, 6$ and 10. The field content is made of a gauge connexion $A = A_\mu dx^\mu$ ($\mu = 0, \ldots, d$) and its gaugino λ, both in the adjoint representation of some gauge group. Transformation laws are determined by Eq. (2) and its Bianchi identity with $\delta^{\,Susy} A = -i(\epsilon\gamma_1\lambda)$, and $\gamma_1 \equiv \gamma_\mu dx^\mu$. We take ϵ commuting so that Eq. (1) holds. To determine $\mathscr{B}^{\rm inv}$, we first make the following observation. In each of the considered dimensions, a Fierz identity shows that $\kappa^\mu \equiv -i(\epsilon\gamma^\mu\epsilon)$ is light-like, that is $\kappa^\mu\kappa_\mu = 0$. We then introduce a vector $\hat\kappa^\mu$ that we normalize so that $\hat\kappa^\mu\kappa_\mu = 1$. Let moreover ι_κ be the contraction operator along κ^μ, so that

$$\delta^{\,Susy} A = -i(\epsilon\gamma_1\lambda)\,, \quad \delta^{\,Susy} F = -d_A\delta^{\,Susy} A\,, \quad \iota_\kappa\delta^{\,Susy} A = 0\,, \quad (\delta^{\,Susy})^2 A = \iota_\kappa F \tag{17}$$

By integrating by parts and with the Bianchi identity $d_A F = 0$, it is straightforward to see that the following expression

$$\mathscr{B}^{\rm inv} = c_n \int {\rm Tr}\, \left(\hat\kappa\, \delta^{\,Susy} A\, \delta^{\,Susy} A\, \delta^{\,Susy} A\, F^{\,n-3}\right) \tag{18}$$

with $\hat\kappa \equiv \hat\kappa_\mu dx^\mu$ and $c_n = \frac{n-2}{3}\binom{n}{2}$ is such that Eq. (15) holds true. Moreover, it provides an off-shell expression, as it is solely based on the geometrical curvature Eq. (2) and its Bianchi identity. The problem thus reduces to that of the elimination of $\hat\kappa$, in order the solution to be bilinear in the supersymmetry parameter. In four dimensions for example, $n = 3$ and the elimination of $\hat\kappa$ in Eq. (18) directly yields the known result [10].

From now on, we focus on the ten-dimensional super-Yang–Mills theory. Its fields content consists of a gauge connection $A = A_\mu dx^\mu$ ($\mu = 0, \ldots, 9$) and a Majorana–Weyl spinor λ, with both λ and $\epsilon \in \mathbf{16}_+$ of $SO(1,9)$. Eq. (15) now reads

$$\delta^{\,Susy}\mathscr{B}^{\rm inv} = -15 \int {\rm Tr}\; \delta^{\,Susy} A\; \delta^{\,Susy} A\; F^4 \tag{19}$$

By demanding removal of the $\hat\kappa$ dependency in Eq.(18), one is naturally led to consider the following solution

$$\mathscr{B}^{\rm inv} = \frac{1}{16} \int {\rm d}^{10}x\, {\rm Tr}\, \left(\varepsilon^{\mu_1\cdots\mu_{10}}(\epsilon\gamma_{\mu_1\mu_2}{}^{\sigma}\lambda)(\lambda\gamma_{\mu_3\mu_4\sigma}\lambda)\, F_{\mu_5\mu_6}F_{\mu_7\mu_8}F_{\mu_9\mu_{10}}\right) \tag{20}$$

Indeed, with the help of some ten-dimensional γ-matrix identities [A], one can check that modulo the equations of motion

$$\begin{aligned}
\delta^{\,Susy}\int {\rm d}^{10}x\, {\rm Tr}\; &\left(\varepsilon^{\mu_1\cdots\mu_{10}}(\epsilon\gamma_{\mu_1\mu_2}{}^{\sigma}\lambda)(\lambda\gamma_{\mu_3\mu_4\sigma}\lambda)\, F_{\mu_5\mu_6}F_{\mu_7\mu_8}F_{\mu_9\mu_{10}}\right) \\
&\approx 15 \int {\rm d}^{10}x\, {\rm Tr}\, \Big(\frac{1}{16}\varepsilon^{\mu_1\cdots\mu_{10}}\, (\epsilon\gamma^\sigma\epsilon)(\lambda\gamma_{\mu_1\mu_2\sigma}\lambda)F_{\mu_3\mu_4}F_{\mu_5\mu_6}F_{\mu_7\mu_8}F_{\mu_9\mu_{10}} \\
&\qquad + \frac{1}{96}\varepsilon^{\mu_1\cdots\mu_{10}}\, (\epsilon\gamma_{\mu_1\mu_2}{}^{\nu_1\nu_2\nu_3}\epsilon)(\lambda\gamma_{\nu_1\nu_2\nu_3}\lambda)F_{\mu_3\mu_4}F_{\mu_5\mu_6}F_{\mu_7\mu_8}F_{\mu_9\mu_{10}}\Big)
\end{aligned}$$

$$= -15 \int \mathrm{d}^{10}x \, \mathrm{Tr} \, \Big(\varepsilon^{\mu_1 \cdots \mu_{10}} \, (\epsilon \gamma_{\mu_1} \lambda)(\epsilon \gamma_{\mu_2} \lambda) F_{\mu_3 \mu_4} F_{\mu_5 \mu_6} F_{\mu_7 \mu_8} F_{\mu_9 \mu_{10}} \Big) \tag{21}$$

It implies that the following expression

$$\mathscr{B} = \int W_{10}^{(0,1)} + \frac{1}{16} \int \mathrm{d}^{10}x \, \mathrm{Tr} \, \Big(\varepsilon^{\mu_1 \cdots \mu_{10}} (\epsilon \gamma_{\mu_1 \mu_2}{}^{\sigma} \lambda)(\lambda \gamma_{\mu_3 \mu_4 \sigma} \lambda) \, F_{\mu_5 \mu_6} F_{\mu_7 \mu_8} F_{\mu_9 \mu_{10}} \Big) \tag{22}$$

solves the supersymmetric part of the consistency equations. We have therefore found the supersymmetric counterpart of the Adler–Bardeen anomaly for the $\mathcal{N} = 1, d = 10$ super-Yang–Mills theory. The result can be easily transposed in $d = 4$ and 6 dimensions. For the case $d = 4$, we recover the result of [10], where the problem is less involved and can easily be solved by inspection over all possible field polynomials.

Acknowledgments

We thank very much G. Bossard for useful discussions. A. M. is grateful to P. Vanhove for his useful advice regarding gamma matrix manipulations. This work has been partially supported by the contract ANR (CNRS-USAR), `05-BLAN-0079-01`. A. M. has been supported by the Swiss National Science Foundation, grant `PBSK2-119127`.

A Ten-dimensional γ-matrix identities

The ten-dimensional γ-matrices satisfy the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ and our convention for antisymmetrization is $\gamma^{\mu_1 \cdots \mu_n} = \frac{1}{n!} \gamma^{[\mu_1} \cdots \gamma^{\mu_n]}$. Both ϵ and λ are chiral, so that we only have to consider a basis of gamma matrices made of

$$\gamma^\mu \, , \quad \gamma^{\mu_1 \mu_2 \mu_3} \, , \quad \gamma^{\mu_1 \cdots \mu_5} \tag{23}$$

Useful identities used to derive (21) are

$$\begin{aligned}
&\gamma^\sigma \gamma^\mu \gamma_\sigma = -8 \gamma^\mu && \gamma^{\sigma_1 \sigma_2 \sigma_3} \gamma^\mu \gamma_{\sigma_1 \sigma_2 \sigma_3} = 288 \gamma^\mu \\
&\gamma^\sigma \gamma^{\mu_1 \mu_2 \mu_3} \gamma_\sigma = -4 \gamma^{\mu_1 \mu_2 \mu_3} && \gamma^{\sigma_1 \sigma_2 \sigma_3} \gamma^{\mu_1 \mu_2 \mu_3} \gamma_{\sigma_1 \sigma_2 \sigma_3} = -48 \gamma^{\mu_1 \mu_2 \mu_3} \\
&\gamma^\sigma \gamma^{\mu_1 \cdots \mu_5} \gamma_\sigma = 0 && \gamma^{\sigma_1 \sigma_2 \sigma_3} \gamma^{\mu_1 \cdots \mu_5} \gamma_{\sigma_1 \sigma_2 \sigma_3} = 0
\end{aligned}$$

as well as $\gamma^{\sigma_1 \cdots \sigma_5} \gamma^\mu \gamma_{\sigma_1 \cdots \sigma_5} = \gamma^{\sigma_1 \cdots \sigma_5} \gamma^{\mu_1 \mu_2 \mu_3} \gamma_{\sigma_1 \cdots \sigma_5} = \gamma^{\sigma_1 \cdots \sigma_5} \gamma^{\mu_1 \cdots \mu_5} \gamma_{\sigma_1 \cdots \sigma_5} = 0$. A generic bi-spinor can be expanded over the basis (23) as

$$\xi \zeta = \frac{1}{16} (\xi \gamma^\sigma \zeta) \gamma_\sigma + \frac{1}{96} (\xi \gamma^{\sigma_1 \sigma_2 \sigma_3} \zeta) \gamma_{\sigma_1 \sigma_2 \sigma_3} + \frac{1}{3840} (\xi \gamma^{\sigma_1 \cdots \sigma_5} \zeta) \gamma_{\sigma_1 \cdots \sigma_5} \tag{24}$$

In particular, the supersymmetry parameter ϵ being commuting, the non-vanishing terms for $\zeta = \xi = \epsilon$ are $(\epsilon\gamma^\sigma\epsilon)$ and $(\epsilon\gamma^{\sigma_1\cdots\sigma_5}\epsilon)$, so that for example $(\epsilon\gamma^\sigma\epsilon)\epsilon\gamma_\sigma = 0$. λ being anticommuting, the only non-vanishing term for $\zeta = \xi = \lambda$ is $(\lambda\gamma^{\sigma_1\sigma_2\sigma_3}\lambda)$. The following identities also turned out to be precious

$$
\begin{aligned}
(\epsilon\gamma_{\mu_1}\lambda)(\epsilon\gamma_{\mu_2}\lambda) &= -\frac{1}{16}(\epsilon\gamma^\sigma\epsilon)(\lambda\gamma_{\mu_1\mu_2\sigma}\lambda) - \frac{1}{96}(\epsilon\gamma_{\mu_1\mu_2}{}^{\nu_1\nu_2\nu_3}\epsilon)(\lambda\gamma_{\nu_1\nu_2\nu_3}\lambda) \\
(\epsilon\gamma^\sigma\lambda)(\epsilon\gamma_{\mu_1\mu_2\sigma}\lambda) &= -\frac{3}{8}(\epsilon\gamma^\sigma\epsilon)(\lambda\gamma_{\mu_1\mu_2\sigma}\lambda) + \frac{1}{48}(\epsilon\gamma_{\mu_1\mu_2}{}^{\nu_1\nu_2\nu_3}\epsilon)(\lambda\gamma_{\nu_1\nu_2\nu_3}\lambda) \\
(\epsilon\gamma_{\mu_1\mu_2}{}^{\nu_1\nu_2\nu_3}\lambda)(\epsilon\gamma_{\nu_1\nu_2\nu_3}\lambda) &= -\frac{21}{4}(\epsilon\gamma^\sigma\epsilon)(\lambda\gamma_{\mu_1\mu_2\sigma}\lambda) - \frac{3}{8}(\epsilon\gamma_{\mu_1\mu_2}{}^{\nu_1\nu_2\nu_3}\epsilon)(\lambda\gamma_{\nu_1\nu_2\nu_3}\lambda)
\end{aligned} \quad (25)
$$

Needless to say, Ulf Gran's GAMMA package [12] was greatly appreciated to derive these identities.

References

[1] J. Wess and B. Zumino, "Consequences of anomalous Ward identities", Phys. Lett. B **37** (1971) 95

[2] R. Stora, in *Progress in gauge field theory*, eds. G 't Hooft *et al.* (Plenum, New York, 1984) 543

[3] L. Baulieu, "Algebraic construction of gauge invariant theories", in Cargese ASI 1983, *Particles and fields*, eds. Basevant *et al.* (Plenum, New York, 1984) 1; L. Baulieu, "Anomalies and gauge symmetry", Nucl. Phys. B **241** (1984) 557

[4] B. Zumino, Y.-S. Wu and A. Zee, "Chiral anomalies, higher dimensions and differential geometry", Nucl. Phys. B **239** (1984) 477

[5] L. Alvarez–Gaumé and E. Witten, "Gravitational anomalies", Nucl. Phys. B **234** (1983) 269

[6] M.B. Green and J.H. Schwarz, "Anomaly cancellations in supersymmetric D=10 gauge theory and superstring theory", Phys. Lett. B **149** (1984) 117

[7] H. Itoyama, V. P. Nair and H.-c. Ren, "Supersymmetry Anomalies And Some Aspects Of Renormalization", Nucl. Phys. B **262** (1985) 317; H. Itoyama, V. P. Nair and H.-c. Ren, " Supersymmetry Anomalies: Further Results", Phys. Lett. B **168** (1986) 78

[8] L. Bonora, P. Pasti and M. Tonin, " Chiral Anomalies In Higher Dimensional Supersymmetric Theories", Nucl. Phys. B **286** (1987) 150

[9] A. Candiello and K. Lechner, "The Supersymmetric version of the Green-Schwarz anomaly cancellation mechanism", Phys. Lett. B **332** (1994) 71, `hep-th/9404095`.

[10] L. Baulieu, G. Bossard and S. P. Sorella, "Shadow fields and local supersymmetric gauges", Nucl. Phys. B **753** (2006) 273, `hep-th/0603248`

[11] M. Henneaux and C. Teitelboim, "Quantization of Gauge Systems", Princeton University Press (Princeton: New Jersey 1992)

[12] U. Gran, "GAMMA: A Mathematica package for performing Gamma-matrix algebra and Fierz transformations in arbitrary dimensions", `hep-th/0105086`